VOLUME – APPLICATION OF INTEGRAL

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Abstract: *The paper presents research results on the formula for calculating the volume of a solid of revolution, some intergral applications - the volume of a solid of revolution in practice, and innovative solutions in teaching integration to practice. Starting with the construction of the formula to calculate the volume of a solid of revolution, we offer two problems that can easily be intergrated into experiential activities to help students carry out practical surveys on measurement and collection of information techniques, stimulating the observational capacity applying the flexibility the formula to calculate the volume of a solid of revolution to solve applied problems. Since then, the authors have developed innovative solutions in the relality-related teaching calculus that not only by supporting teachers who can guide students to turn dry calculus problems into practical problems in life explicitly, but also helps students have a different perspective, more interest and passion with countless practical applications waiting for students to discover and accept.*

Keywords: *Integral, application of integral, volume of a solid of revolution amount of propellant, K56 bullets, sphere-shaped high-tech vegetable greenhouse in Hoi An, integral teaching solutions, history of integrals, extracurricular activities, interdisciplinary.*

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1. INTRODUCTION

Mathematics is a fundamental science like other sciences in the natural sciences; mathematics is closely related to practice and is widely applied in many fields of science and technology, production, and social life today. As a result of the development dynamics of the large industry, the explosive speed of science and technology is reflected in new milestones of achievements as well as a highly practical application of capabilities. And mathematics is one of the tools of these sciences. In recent years, the educational economy has changed the tendency to develop pedagogical thinking in the direction of "Strengthening the application of Mathematics in practice" whose broad content is "Learning goes hand in hand with practice, combined with productive labor, the theory associated with practice, school education combined with family and social education. Integral is a mathematical concept that has a wide range of applications in probability statistics, physics, mechanics, astronomy, medicine, and technological sectors including shipbuilding, vehicle manufacturing, machining, and aviation. Basic integral applications were also included in the textbook curriculum, but most high school students stop at using formulae to solve internal mathematical issues and have no idea how to apply them to other topics or to real life. As a result, we want to discuss one of the practical applications of integrals - the volume of a solid of revolution, from which students may learn more engaging and passionate methods to acquire math, with endless practical applications awaiting their discovery and acceptance.

2. CONTENT

2.1. Formula for calculating the volume of a solid of revolution:

[2] Consider the graph $y = f(x)$ in Fig. 1. If the upper half - plane is rotated about the x -axis, then each point on the graph has a circular path, and the whole graph sweeps out a certain surface, called a surface of revolution.

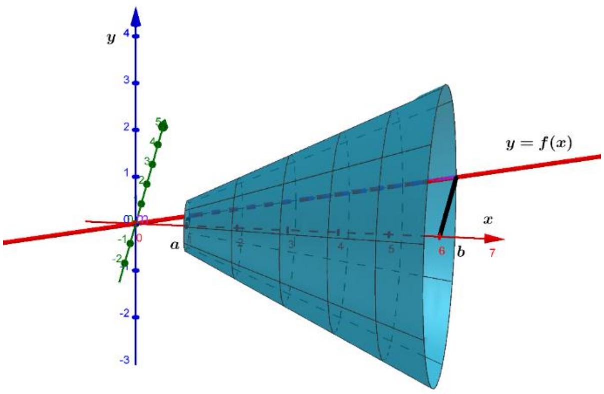

Figure 1. The graph $y = f(x)$ - The plane region is bounded by the graph, the x - axis, $x = a$ and $x = b$ sweeps out a solid of revolution.

To calculate the volume of this solid, we first approximate it as a finite sum of thin right circular cylinders, or disks (Fig. 2). We divide the interval $[a;\ b]$ into equal subintervals, each of length Δx. Thus, the height h of each disk is Δx (Fig. 3). The radius of each disk is $f(x_i)$, where x_i is the right - hand endpoint of the subinterval that determines that disk. If $f(x_i)$ is negative, we can use $|f(x_i)|$.

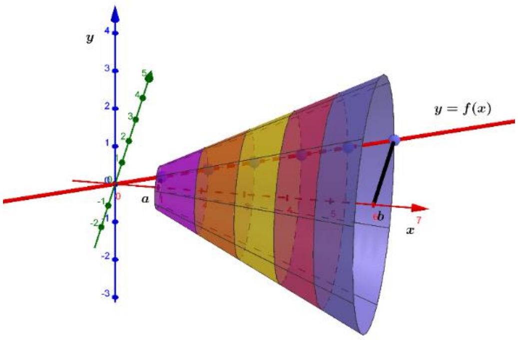

Figure 2. The graph y = f(x) is divided the interval into equal subintervals, each of length.

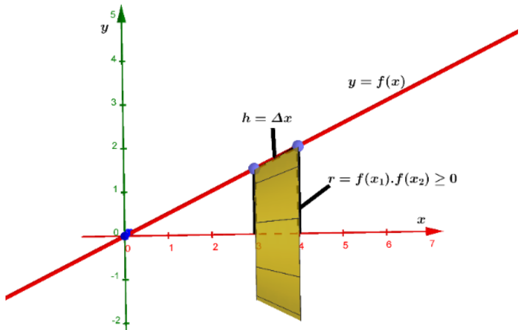

Figure 3. The height of each disk

Since the volume of a right circular cylinder is given by $V = r^2h$ or volume equals the area of the base times height, each of the approximating disks has volume $\pi|f(x_i)|^2\,\Delta x = \pi[f(x_i)]^2\Delta x$ (Squaring makes use of the absolute value unnecessary).

The volume of the solid of revolution is approximated by the sum of the volumes of all the disks:

$$V \approx \sum_{i=1}^{n} \pi[f(x_i)]^2 \Delta x$$

The actual volume is the limit as the thickness of the disks approaches zero, or the number of disks approaches infinity:

$$V = \lim_{n\to\infty} = \sum_{i=1}^{n} \pi[f(x_i)]^2 \Delta x = \int_a^b \pi\left[f\left(x\right)\right]^2 dx$$

That is, the volume is the value of the definite integral of the function $y = \pi\left[f\left(x\right)\right]^2$ from a to b .

For a continuous function f defined on $[a;\ b]$, the volume V of the solid of revolution obtained by rotating the area under the graph of f from a to b about the x -axis is given by

$$V = \int_a^b \pi\left[f\left(x\right)\right]^2 dx$$

2.2. Some application of integral:

The expanding use of mathematic outside of mathematics will aid in clarifying the function of the subject's special tool, the billboard, in the transition from warfare to normalcy. It is not only about numbers; it is about supporting humanity in achieving major progress, as well as assisting the younger generation in gaining access to, participating in, and encouraging the country's development and growth.

2.2.1. Integral application to calculate the amount of propellant contained in K56ammunition

In the previous war of liberation, K56 ammunition (produced by China in 1956 and Vietnam known as K56 ammunition) was very popularly used because of its convenience and high efficiency the object is the AK-47 submachine gun (a type of gun designed by the Soviet Union in 1947).

Although the world is currently in a period when military science and technology develops like a storm, the enemy can use many types of modern weapons, but to solve the battlefield, it is necessary to use infantry. Therefore, in the future war, K56 ammunition is still the most used ammunition and will bring high efficiency.

[4] A bullet has a structure of 4 parts: the shell, the fire particle, the primer, and the warhead.

The shell is the largest part, helping to connect the other parts of the bullet, and especially this is also the part that stores and protects the propellant. The shell consists of 4 parts: the shell body, the shell neck, the shell shoulder, and the bottom of the shell.

The shell body has a cylindrical structure, which is the main part used to store the bullet's propellant.

In this article, we would like to introduce readers to the application of integrals to calculate the volume of propellant contained in the K56 shell most accurately.

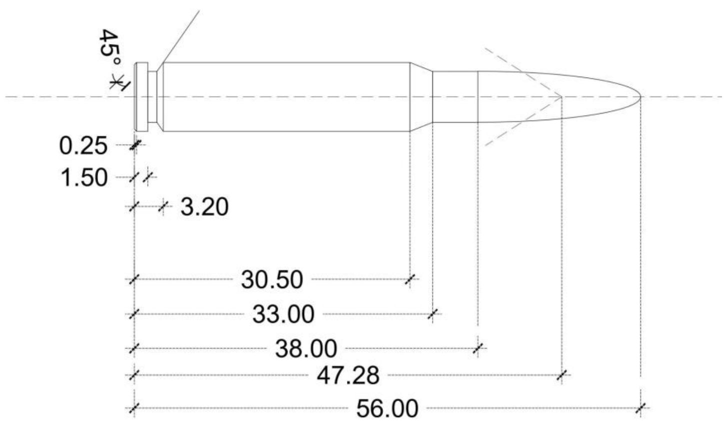

Figure 4: Dimensional parameters on the shell of the K56

Applying formula: $V = \int_a^b \pi\left[f(x)\right]^2 dx$

We will set up the function. In general, the constant function $y = a$.

The K56 bullet of the AK - 47 guns has a cylindrical shape with a body length: 27.3mm and a diameter of 11.35mm.

From there we denote:

Radius: $r = 5,675mm$

Height: $h = 27,3mm$

Then in the coordinate system, Oxy the cylinder has the form of a straight line with a function of $y = 5,675$.

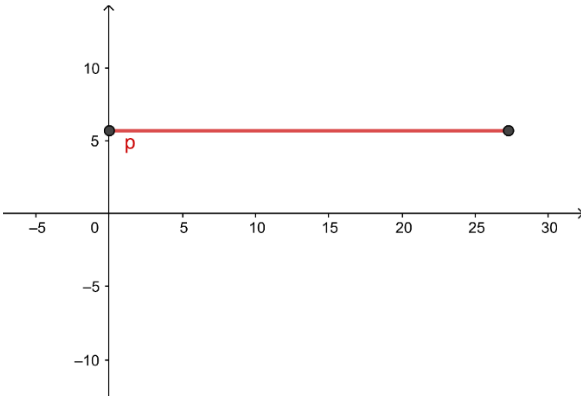

Figure 5: K56 bullet graph in Oxy coordinate system

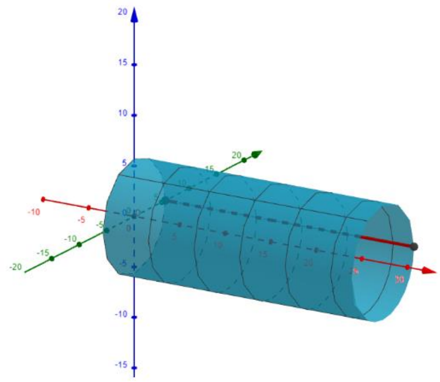

Figure 6: K56 bullet graph in Oxyz coordinate system

Applying the formula for volume, we have:

$$V = \int_0^{27,3} \pi(5,675)^2 dx = \int_0^{27,5} 32,205625\pi dx = 32,205625\pi x \,\Big|_0^{27,5} = 2782,36626\left(mm^3\right)$$

2.2.2. Sphere-shaped high-tech greenhouse for growing vegetables in Hoi An

Figure 7: Sphere-shaped high-tech greenhouse for growing vegetables in Hoi An

Inside the 1,000 square meters, the spherical greenhouse is a climate control system, multi-story cultivation and Israel's smart irrigation system. Operating since April, VinEco Nam Hoi An, belonging to Vinpearl Nam Hoi An complex is a large farm (13.2 hectares) applying modern and smart technology, ranking the third after Tam Dao farm.

Vinh Phuc and Long Thanh - Dong Nai belong to Vingroup. Not only is it a high-tech agricultural greenhouse, providing the products of vegetables, organic fruits, VinEco

Nam Hoi An but also opens daily to welcome the visitors, visiting the organic agriculture model which applies smart technology.[1]

Dome greenhouse covers an area of 1.000 square meters; the structure is a large sphere with 36 meters in diameter and 14 meters in height. Inside of it is France's climate control system (temperature, humidity, light) and multi-story cultivation- Sky Green of SkyUrban company (Singapore). [1].

To calculate the required air-conditioning capacity for a Sphere-shaped high-tech greenhouse for growing vegetables in Hoi An, we must calculate the volume of the greenhouse.

Using the formula: $V = \int_a^b \pi \left[f(x) \right]^2 dx$

We will set up the function.

In general, the circle with center $(h;k)$ and the radius is r, equation: $(x-h)^2 + (y-k)^2 = r^2$

Sphere-shaped high-tech greenhouse for growing vegetables in Hoi An covers an area of 1.000 square meters, a structure is a large spherical shape with 60 meters in diameter and 14 meters in height.

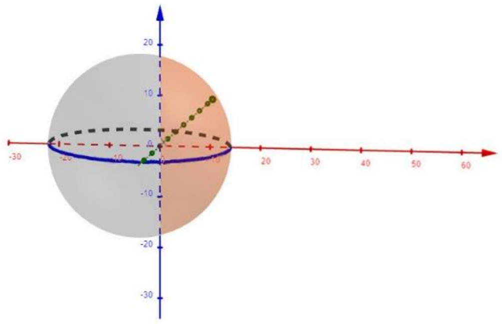

Figure 8. The sphere in the Oxy coordinate system.

In conclusion, we denote:

Radius: $r = 18m$

Height: $h = 14m$

Then in Oxy coordinate system this circular sphere with center $(-4;0)$ and radius $18m$.

$$(x+4)^2 + y^2 = 18^2$$

$$\Rightarrow y = \sqrt{18^2 - (x+4)^2}$$

Applying the formula of volume, we have:

$$V = \int_0^{14} \pi\left(\sqrt{18^2-(x+4)^2}\right)^2 dx = \int_0^{14} \pi\left(-x^2 - 8x + 308\right)dx$$

$$= \pi\left(-\frac{1}{3}x^3 - 4x^2 + 308x\right)\Bigg|_0^{14} = \frac{7840}{3}\pi\left(m^3\right)$$

Calculating the required air-conditioning capacity for Sphere-shaped high-tech greenhouse for growing vegetables in Hoi An.

The required capacity for a room= Room volume * 200 BTU (equivalent to 200 BTU/ m^3)

Inside, BTU is a British Thermal Unit, which is used to measure the volume of heating or cooling devices, can exchange 9.000 BTU = 1 HP (1 Horse Power).

The required air-conditioning capacity for Sphere-shaped high-tech greenhouse for growing vegetables in Hoi An:

$$\frac{7840}{3}\pi.200 = 1642005,76\left(BTU/m^3\right) \approx 182,445\left(HP\right)$$

In conclusion, the required air-conditioning capacity for a Sphere-shaped high-tech greenhouse for growing vegetables in Hoi An is $182,445(HP)$.

2.3. Innovative solutions in teaching integrals associated with the practice

[3] The solutions given are aimed at the purpose, feasibility, and effectiveness of the teaching and integration process in high schools.

Measure 1: When teaching integration, we must take advantage of every opportunity to clarify the history of the birth and the practical origin of knowledge.

Integral has gone through many ups and downs, historical events, stemming from the reality of human life, through layers of stages, to become what it is today. When students understand the practical origins of calculus, then they will easily grasp knowledge, better understand the problems they are approaching, thereby increasing their enthusiasm for math.

Some of the contents in the textbooks show concepts and definitions derived from practice, but it is not clear which ones originate from practice, but only definitions and concepts are given. Therefore, for students to understand more deeply, in the additional reading section, the author of the Textbook has given the origin and meaning of their birth such as the article "History of integral calculus" "The product approximation". Calculus and the concept of total integrals" ("Additional Reading" Textbook of Advanced Calculus 12 pages 154)…

As in the "Did you know" section of the 12^{th}-page Calculus Textbook, page 122 shows that: to define integrals, these mathematicians did not use the concept of limits. Instead, they say "the sum of an infinitely large number of infinitely small terms". For example, the area of a curved ladder is the sum of an infinitely large number of areas of infinitely small rectangles. Based on that base has been calculated accurately many areas of flat shapes and volumes of objects.

Teachers can introduce students in the process of teaching the application of integrals about the story that Archimedes also found the area of a circle by his method. This was the first model of integral calculus, whereby he found an approximation of the number π between two fractions $3\frac{10}{71}$ and $3\frac{1}{7}$.

Measure 2: Take practical examples and explain the meaning.

In the process of teaching integrals, teachers need to give practical examples so that students can easily grasp and relate, avoiding the situation of learning rice, rote learning, etc. As can be applied, Integrate into the volume of objects (infusion rate of pouring funnel, amount of drug in pill, amount of propellant in the cartridge, the volume of cream in a cone, vase, barrel of wine,...), or in problems of calculating speed, distance (when the vehicle is moving, brake, find the distance to go after,...), or calculate the area (calculate the area of the fish pond, the area of the garden, the welcome gate. according to ellipse, parabola, rim… even patterns that don't follow a certain shape…) Teachers can learn and introduce them to students or suggest directions to help them research and explore on their own.Teachers not only guide students to relate knowledge to practice on their own but also help students put theory into practice.

The teacher can give an example of building a tent to celebrate the 3 – 26th founding day of the Ho Chi Minh Communist Youth Union so that students can calculate the lowest cost not only in the tent but also decorate it right away in my room. The school delegation asked students to decorate the tent on the ABCD rectangular board on the Parabolic panel as shown. Know that the cost to decorate a square meter of the board is 100 thousand VND. Find the lowest cost to complete the decoration (round to thousands)?

Solution

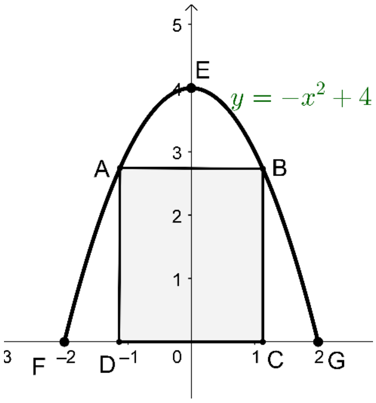

Figure 9.

The graph y = - x² + 4 and the ABCD rectangular board on the Parabolic panel

Suppose the Parabola is $(P): y - ax^2 + bx + c$.

Then (P) passes through three points:

$$E(0;4); F(2;0); G(-2;0) \Rightarrow \begin{cases} a = -1 \\ b = 0 \\ c = -4 \end{cases} \Rightarrow (P): y = -x^2 + 4$$

Set $CD = 2x, 0 < x < 2 \Rightarrow C(x;0) \Rightarrow BC = -x^2 + 4$

$$S_{Trangtri} = \int_{-2}^{2} \left(-x^2 + 4\right) dx - 2x\left(-x^2 + 4\right) = 2x^3 - 8x + \frac{32}{3} = g(x)$$

The cost to decorate is: $C = 100000.S_{Trangtri} = 100000.g(x)$.

We have: $g'(x) = 6x^2 - 8 = 0 \Leftrightarrow x = \frac{2}{\sqrt{3}} \in (0,2)$ so we have the following variation table:

x	0		$x = \frac{2}{\sqrt{3}}$		2
$g'(x)$		$-$	0	$+$	
$g(x)$			$\frac{96 - 32\sqrt{3}}{9}$		

Figure 10

From the variation table we have $C \geq 100000.\frac{96 - 32\sqrt{3}}{9}$. The equal sign occurs when $x = \frac{2}{\sqrt{3}}$.

So, min $C = 100000.\frac{96 - 32\sqrt{3}}{9} \approx 451000$ (dong).

Measure 3: Extracurricular activities related to integrals.

In the current context of textbooks and program distribution, organizing extracurricular activities for mathematics in general and calculus, in particular, is an appropriate and highly feasible solution. Along with the regular lesson, extracurricular activities contribute to stimulating learning for students; supplement and expand the knowledge, contribute to self-development and self-improvement; Good health; foster students' aptitudes and creative talents; Training students many necessary life skills such as communication skills, handling situations, teamwork, respecting individual differences, etc. Through extracurricular activities, teachers discover and foster talent in mathematics, which shows in mathematical activities, their ability to detect and solve mathematical problems arising in theory as well as in practice.

Nowadays, there are many types of extracurricular activities applied by schools, including sports activities, art activities, organizing communities, joining clubs, performing arts, etc. Extra-curricular activities with integral themes can take place in many forms. For example, an extracurricular activity session at Banh It Tower – Binh Dinh.

The tower has two doors; each door is shaped like a parabolic, located on the same axis (East-West direction). Students' task is to visit and find out how many meters apart the two doors are, the height and width, connecting doors. Using these data, write an essay that calculates the volume of the limited aisle space between two doors.

Figure 10. Banh It Tower – Binh Dinh.

Considering one door of Banh It Tower, set the axis system as below figure.

It is easy to find the equation of the parabola: $y = -16x^2 + 4$.

Area of the parabola: $S = \int_{-0,5}^{0,5} \left(-16x^2 + 4\right) dx = \frac{8}{3}$.

The volume of the limited aisle space between the two doors:

$$8.\frac{8}{3} = \frac{64}{3}\left(m^3\right).$$

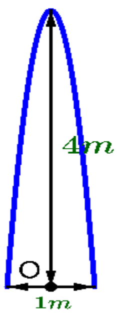

Figure 11. The graph $y = -16x^2 + 4$

Measure 4: Embrace the spirit of interdisciplinary so that students can easily relate and better understand the roles of integral

This solution aims at applying integrals to other subjects in the school. These activities can be conducted during mathematics lesson, also conducted by other teachers in teaching between mathematics and other subjects, particularly in the Faculty of Natural Sciences. From there, it shows the relationship, diversity, and interdisciplinary spirit of integrals.

In the scope of high school, good interdisciplinary performance is a principled requirement in teaching in general and in teaching mathematics in particular. The connection between mathematics and other subjects is made mainly by using the knowledge and skills of mathematics, solving mathematical modeling of situations appearing in the sciences natural learning, even though the situations are simple and generally simulated.

Comprehensive exercises with interdisciplinary content are situations in other subjects (physics, chemistry, biology, etc.) requiring us to use integrals to solve. In Physics at grade 10 we use integrals to calculate distances as below:

A car starts to accelerate at a constant speed $v_1\left(t\right)=7t\ \left(m/s\right)$.When going, the driver detects an obstacle and applies the brakes, the car continues to slow down uniformly with acceleration $a=-70\ \left(m/s^2\right)$. Calculate the distance $S\ \left(m\right)$ the vehicle's range from the moment it starts to change gears until it comes to a complete stop.

Solution

In the first 5 seconds, the car covers a distance $S_2=\int\limits_0^5 7tdt=\frac{7}{2}t^2\Big|_0^5=87,5\left(m\right)$.

Since the break $v_2=\int\left(-70\right)dt=-70t+c$.

When the car starts to brake $t=0$ then $v_2=35\left(m/s\right)$ deduce $35=-70.0+c\Rightarrow c=35$.

When the car comes to a complete stop $v_2=0\Rightarrow -70t+35=0\Rightarrow t=\frac{1}{2}$.

Distance traveled by car since applying the brake $S_2=\int\limits_0^{\frac{1}{2}}\left(35-70t\right)dt=\frac{35}{4}\left(m\right)$.

The distance traveled by the car from the moment it starts to change gears until it comes to a complete stop is $S=S_1+S_2=96,25\left(m\right)$.

3. CONCLUSION

The article mentions how to build a formula to calculate the volume of a solid of revolution, given some practical problems that are very close to high school students and innovative solutions in intergral education, associated with practice. In addition to building a formula to calculate the volume of a solid of revolution in an intuitive and specific way so that students can understand the essence and deepen the formula, teachers should provide practical application problems that are related to each other, nearby subjects or group activities with real-life experiences to provide a way to solve problems. Moreover, it helps students practice their thinking, creativity and teamwork skills effectively . From the above article, we would like to propose interdisciplinary integration to help students have a positive vision and feel the beauty of Mathematics.

REFERENCE

1. Anh Tuyet (2018), *Sphere-shaped high-tech greenhouse for growing vegetables in Hoi An*, retrieved from https://vnexpress.net.
2. Marvin L. Bittinger, David J. Ellenbogen, Scott A. Surgent (2012), *Calculus and its applications*,Tenth Edition, United States of America, tr.522.
3. Mai Thi Ngoan (2010), "Teaching primitives and integrals in the direction of enhancing the ability to apply mathematical knowledge into practice for high school students", *Master thesis in education, theory and practice*, Math teaching methods, Vinh University.
4. VOER. (n.d.), Introduction Several types of infantry weapons. Vietnam OER, Open Vietnam Educational Resources, retrieved from https://voer.edu.vn.

THỂ TÍCH KHỐI TRÒN XOAY - ỨNG DỤNG CỦA TÍCH PHÂN

Tóm tắt: *Bài báo trình bày kết quả nghiên cứu về công thức tính thể tích khối tròn xoay, một số ứng dụng của tích phân – thể tích khối tròn xoay trong thực tiễn và các giải pháp đổi mới trong dạy học tích phân gắn với thực tiễn. Xuất phát từ việc xây dựng dựng công thức tính thể tích khối tròn xoay, chúng tôi đưa ra hai bài toán có thể dễ dàng lồng ghép vào hoạt động trải nghiệm giúp học sinh có những khảo sát thực tế về đo lường thu thập các thông số kĩ thuật, kích thích khả năng quan sát, vận dụng linh hoạt công thức tính thể tích khối tròn xoay để giải quyết các bài toán ứng dụng. Từ đó nhóm tác giả đưa ra các giải pháp đổi mới trong dạy học tích phân gắn với thực tế không chỉ hỗ trợ giáo viên có thể định hướng cho học sinh biến những bài tích phân khô khan thành những vấn đề thiết thực trong cuộc sống một cách tường minh, mà còn giúp cho học sinh có góc nhìn khác, hứng thú và say mê hơn với vô vàn ứng dụng trong thực tiễn đang chờ đón học sinh khám phá và đón nhận.*

Từ khóa: *Tích phân, ứng dụng của tích phân, thể tích khối tròn xoay, lượng thuốc phóng, đạn K56, nhà kính trồng rau công nghệ cao hình quả cầu tại Hội An, giải pháp dạy học tích phân, lịch sử tích phân, hoạt động ngoại khóa, liên môn.*